The Diffusion of Services

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Abstract

The service sector has expanded in recent decades. One important influence on its growth and long-term profits is *customer attrition*, which can occur at the category level (*disadoption*) or between firms (*churn*). Yet the literature has rarely modeled how services penetrate a market and has not evaluated attrition's effect on growth.

We combine diffusion modeling with a customer relationship approach to investigate the influence of attrition on growth in service markets. In particular, we model the effects of disadoption and churn on evolution of a category and on growth of individual firms in a competitive environment.

We show how neglecting disadoption can bias parameter estimation and especially market potential. We also derive an expression for the customer equity of a growing service firm and apply it to valuation of firms operating in competitive industries. Our results for six of seven firms in four service categories are remarkably close to stock market valuations, an indicator for the role of customer equity in valuations of growing service firms.

INTRODUCTION

Numerous new products introduced during the last few decades are services rather than durable goods. Now widely used services such as cellular phones, satellite radio, and financial services such as direct banking were not available before 1980. The growth of the Internet also drove the offering of many new services, including instant messaging, shopping portals, and online brokerages. Indeed, the service sector in the U.S. employs most of the nation's work force, is responsible for more than 80% of the gross domestic product (GDP), and is growing considerably faster than the goods sector (Zeithaml and Bitner 2003; U.S. Bureau of Economic Analysis (BEA) 2003).

A considerable influence on the market growth of a new service is *customer attrition*. Beginning with the initial stages of penetration into a market, there are customers who leave the service: They switch to competitors or, alternatively, leave the category. In this sense, the growth of a new service is similar to a leaking bucket—there is an inward flow of adopters and a concurrent outward flow of customers who leave. Customer attrition (and its complement, customer retention) has gained considerable attention from managers and researchers in recent years after demonstrations of the relationship between a firm's customer retention rate and its long-range profits (Reichheld 1996). Customer retention is a basic component in the computation of *customer lifetime value* (Kumar and Shah 2004) and its antecedents and consequences for service firms have been the focus of much research attention in recent years.

Despite these facts, the literature dealing with the evolution of markets for new products has dedicated little effort to defining and modeling the effect of attrition on the growth of service markets. The diffusion-modeling literature, which has been the main thrust of research on growth of new markets (Mahajan, Muller, and Wind 2000; Bass 1969), has generally focused on the growth of category-level markets for single-purchase durable goods and has not dealt with services and customer attrition. Studies examining the growth of competitive markets have mostly focused on the competition for acquiring new customers from the remaining market potential and not on interfirm switching (Krishnan, Bass, and Kumar 2000; Kalish, Mahajan, and Muller 1995; Teng and Thompson 1983).

The goal of this paper is to provide a framework that enables researchers to understand how the dynamics of customer attrition affect the growth of markets for services and to examine the resulting consequences for management. We present a multifirm model that captures the complex dynamics of customer acquisition and retention during a service firm's growth: In any given period, a firm can acquire customers from the pool of nonusers (which includes new customers as well as customers who disadopted the category in the past) and also acquire customers who switch from a competitor (known as "churn"). Alternatively, the firm can lose customers to a competitor (churn) or to disadoption from the category. While the dynamics are not trivial, our model is relatively simple and enables an in-depth analysis of the growth of services.

We demonstrate the implications of our approach in two ways. We start with a simpler model that focuses on category-level growth. This model allows us to consider how category-level attrition (disadoption) affects the growth of service categories. We show that neglecting attrition and using the classic diffusion approach—an approach intended originally for durables but widely used in service markets—can create considerable bias in parameter estimations and, more seriously, in estimates of market potential.

We then construct a model of a competitive brand-level market that considers interfirm churn as well as category disadoption. We use this model to calculate the *customer equity* for service firms. Customer equity, which represents the sum of the lifetime values of a firm's customers, has emerged in recent years as a key marketing measure that can be used to assess the return on marketing activities and the value of firms (Kumar and George 2007; Peppers and Rogers 2005; Gupta, Lehmann, and Stuart 2004; Rust, Zeithaml, and Lemon 2000). Our model for calculations of customer equity enhances existing aggregate approaches in two aspects: it provides for brand-level analysis and it incorporates attrition both in customer lifetime value and in the growth function. Hence, our model is especially well suited to cases such as cellular phone service in which inter-firm customer churn is an integral part of the growth process. Similar to Gupta, Lehmann, and Stuart (2004), we compare our customer equity measures with estimations of firms' values by the stock market and show that, in six of seven cases in four service categories, our estimations are notably close to the stock market valuations for the firms. We also show that neglecting to take attrition into account leads to a considerable underestimation of the value of the customer base.

The rest of this paper continues as follows. We first briefly review the relevant literature concerning the empirical and theoretical aspects of attrition and service diffusion. Next, we present our model at the category level and its underlying assumptions and study the influence of disadoption on market growth. We then explore the competitive model and calculate the customer equity of seven brands of services in markets for mobile phones, online brokerages, online book retailers, and satellite radio providers and discuss their customer equity relative to a model without attrition and to stock market valuations. We conclude by discussing theoretical and practical implications.

DIFFUSION AND ATTRITION

Services have traits in common with both durable goods and fast-moving consumer goods. Like sellers of fast-moving consumer goods, service providers depend on repeat purchases for commercial success. The growth of the market for fast-moving consumer goods is usually attributed to advertising, promotion, and consumer trials; therefore, studies of such goods usually rely on frameworks such as stochastic choice models. Purchase decision-making for services, on the other hand, is governed by internal communication mechanisms such as word of mouth and imitation (Wangenhein and Bayon 2004; Murray 1991). In this sense, services are similar to durable goods. However, a major difference between durable goods and services is the existence of the outward flow of customers, *customer attrition*, wherein a customer decides to terminate the relationship with the provider.

Attrition is mainly relevant to services that entail regular repurchases and in which customers develop long-term relationships with service providers (Berry 1999). Hence we focus on *continuous service* encounters that are characterized by some kind of longitudinal customer-firm relationship (Bolton and Lemon 1999). Examples include cable television, telephone, online services, and financial services.

Attrition (and its complement, retention) has become an important subject when analyzing the relationships of firms with their customers. Since the early 1990s, the business literature has begun to focus on retention rate as a major component of firms' long-term success (Reichheld 1996). In the academic literature, one can see increasing attention being paid by marketing researchers to the antecedents and consequences of customer retention (Lewis 2004; Thomas, Blattberg, and Fox 2004; Lemon, White, and Winer 2002).

Since we approach attrition from both category and firm levels, two research schools are of interest: diffusion of innovation modeling and competitive dynamics during market growth. Regarding the former, customer attrition has not been formally integrated into models of the diffusion of innovation. The diffusion literature has generally focused on the category level and modeled the diffusion of services as if they were durable goods for categories that include cellular phones (Krishnan, Bass, and Kumar 2000), land-line phones (Jain, Mahajan, and Muller 1991), cable television (Lilien, Rangaswamy, and Van den Bulte 2000), and online banking (Hogan, Lemon, and Libai 2003).

Some diffusion-related studies have analyzed long-run effects that extend beyond the original purchase, including studies of replacement of worn-out units with new ones and multiunit ownership (Kamakura and Balasubramanian 1987; Steffens 2003; Ratchford, Balasubramanian, and Kamakura 2000), the growth of successive generations of products (Mahajan and Muller 1996; Norton and Bass 1987), and trial-repeat models for pharmaceuticals (Hahn et al. 1994; Lilien, Rao, and Kalish 1981). In spite of their long-range views, however, these models focused on goods, not on services, and do not relate specifically to attrition.

Some diffusion studies have looked at growth in a competitive market (see Chatterjee, Eliashberg, and Rao (2000) for a review). These studies generally investigated one of two scenarios. One is the case of a saturated market that is usually described using a Lanchester formulation. In this scenario, the *total* number of customers remains constant and the firms compete directly to gain each other’s customers (Chintagunta and Vilcassim 1992). The other scenario, which is usually described using a Vidale-Wolfe formulation, typically assumes that firms compete for the remaining market potential of nonadopters; the models do not relate to direct transfers of customers between firms (Krishnan, Bass, and Kumar 2000; Givon, Mahajan, and Muller 1995; Parker and Gatignon 1994; Eliashberg and Jeuland 1986). Thus there is a need for an approach that explicitly incorporates both customer switching and competitive growth.

With the general term *attrition* we denote any case of a customer who terminates a relationship with a service provider. In competitive environments like those for mobile telephone, cable and satellite television, e-banking, and other subscriber-based services, attrition is an important operational measurement that is monitored regularly by service providers. Attrition rates are influenced by customer satisfaction (Bolton 1998), competitive pressure (Oliver 1999), switching costs (Burnham, Frels, and Mahajan 2003), and customer information on alternatives (Capraro, Broniarczyk, and Srivastava 2003).

Most of the literature on customer profitability has focused on cases of *churn* (an exiting customer is acquired by a competitor). However, customers also can disadopt—leave the service category altogether (Hogan, Lemon, and Libai 2003). Empirical evidence for service industries suggests that many customers stop using a new service during the growth stage (Sarel and Marmorstein 2003; Kramer 2002; Reichheld and Schefter 2000), a phenomenon that may intensify as consumers are pressured by firms to adopt new technologies (Meuter et al. 2005). Thus, attrition consists of churning and disadopting customers and the attrition rate is the sum of the churn rate and the disadoption rate.

There are primarily two ways in which marketing modelers have considered customer attrition. *Lost-for-good* attrition occurs when the customer is not expected to return in the foreseeable future. Due to their simplicity, lost-for-good assumptions frequently have been used for lifetime value calculations (Gupta, Lehmann, and Stuart 2004; Berger and Nasr 1998). Use of the lost-for-good approach has been criticized by Rust, Lemon, and Zeithaml (2004), who proposed an alternative *migration* approach in which the customer leaves for a limited time—possibly to a competitor—and then may return.

An interesting question relates to the kind of attrition that disadoption represents. Hogan, Lemon, and Libai (2003) modeled both a case in which the customer is lost for good once s/he disadopts and a case in which, after leaving the service, the customer comes back during the growth process. The latter case is probably more realistic for most innovative services. Longitudinal improvements, especially in terms of the service's price and quality, coupled with reduced uncertainty and growing social pressure to adopt, allow a customer who is returning to a previously dropped service to form reasonable expectations about the service. Indeed, one reason for considerable investments in online banking in the late 1990s was the realization that the low utility of the service in its initial form was driving attrition. When online banking became more user-friendly and functional, some of those who had tried it earlier and were disappointed eventually returned (Monahan 2000).

MODEL OF CATEGORY-LEVEL SERVICE GROWTH WITH ATTRITION

In this section we introduce a category-level model for services that incorporates disadoption and use the model to understand how the disadoption rate affects the growth of a new service category. As noted in the previous discussion, two options are available for modeling attrition in general and disadoption in particular. The first is lost-for-good disadoption in which disadopting customers will never join the service. In the second, a disadopter may rejoin the service at a later date.

From a dynamic modeling point of view, the lost-for-good option is problematic since the constant disadoption leads to a zero level of adoption in the long run regardless of the values of the rest of the parameters. This fact is inconsistent with classical diffusion approaches as well as with empirical data. In addition, as mentioned earlier, anecdotal evidence supports the option of a customer who might eventually return. Note that, while theoretically a customer might return immediately after disadoption, re-adoption typically takes quite a while because the individual's return is again subject to the diffusion process. Hence, in our model, and consistent with calls to take customers' eventual return into account when modeling attrition (Rust, Lemon, and Zeithaml 2004), we assume that disadopting customers can rejoin.

When giving disadopters the possibility to return, one must decide how to model the external and internal influences on returning disadopters compared with first time adopters. There is no empirical evidence that supports a clear assumption on this point, and yet studies of customers' lifetime values commonly assume uniform influence of acquiring new customers and of regaining former ones (Villanueva and Hanssens 2007). Recent approaches that have examined the recapturing of lost customers have also avoided making assumptions about reactions to marketing efforts by new versus recaptured customers (Thomas, Blattberg, and Fox 2004). Given the absence of detailed information on return probabilities, we do not distinguish between these two types of customers in the potential market. In the last section of the paper, we discuss the potential implications of relaxing this and other assumptions.

Following the preceding, then, let $N(t)$ be the users or subscribers, m the market potential, p and q the external and internal parameters, and δ the disadoption rate.

The diffusion of the new service is thus given by the following equation:

$$\frac{d\,N(t)}{d\,t} = p\big(m - N(t)\big) + \frac{q(1-\delta)N(t)}{m}\big(m - N(t)\big) - \delta\,N(t) \quad . \tag{1}$$

Note that at each point of time we assume that only those who did not disadopt spread positive word-of-mouth communications about the product. Thus, while the degree of the word-of-mouth promotion by those customers who are retained is the same (*q*), the effective word-of-mouth impact that takes into account the opportunity for that influence to take place is reduced due to the disadoption rate δ from $\frac{qN(t)}{m}$ to $\frac{q(1-\delta)N(t)}{m}$. Since disadopters return to the market potential, the remaining market potential remains *m-N(t)* and is not affected by the disadoption.

Equation 1 is a first-order quadratic differential equation. Using the initial condition *N(0) = 0*, we can integrate Equation 1 to arrive at the following solution (the derivations are given in Appendix A – available at the *JMR* website):

(2) $$N(t) = \frac{\overline{m}(1-e^{-(\overline{p}+\overline{q})t})}{1+(\overline{q}/\overline{p})e^{-(\overline{p}+\overline{q})t}}.$$

(3) $\overline{m} \equiv m\frac{\Delta+\beta}{2q(1-\delta)}$, $\overline{p} \equiv \frac{\Delta-\beta}{2}$, $\overline{q} \equiv \frac{\Delta+\beta}{2}$, $\beta \equiv q(1-\delta) - p - \delta$, and $\Delta \equiv \sqrt{\beta^2 + 4q(1-\delta)p}$.

Interestingly, the market penetration curve (2) has the same functional form as the Bass equation (1969) but with different parameters: $\overline{q}$, $\overline{p}$, and $\overline{m}$ instead of *q*, *p*, and *m*. It can be easily shown that both β and Δ decrease with respect to δ, hence $\overline{m} < m$, $\overline{p} > p$, $\overline{q} < q$, and all three parameters are positive. When the disadoption rate δ is zero, it is easy to see that Equation 2 converges with the Bass diffusion function.

In order to assess the reliability and identification of the model we performed a series of simulations where we generated data with our model, estimated its parameters and observed whether our model returns the correct parameters. We generated a full factorial combination of parameter values in typical ranges and performed nine sets of 64 simulations each: one with clean data and the others where we added a random noise to the data - each point of data was perturbed with a normally distributed noise term, whose standard deviation determined the noise level. We checked several noise levels, and two noise generation mechanisms – one where the noise term had a fixed standard deviation (normalized as a percentage of market potential), and relative noise, where the standard deviation of the noise term was proportional to the value of each data point. Although, as explained below, in the empirical estimation we treat δ as exogenous, the simulations estimated the disadoption rate from the data. The simulation results, available in Appendix B (available at the *JMR* website), indicate that even in the high noise level, the large majority of the parameters where correctly estimated, for example in the (relative) 6 percent noise level, 93% of the parameters were not significantly different from the original parameters at the at the 5% level.

An important implication of Equation 2 is that the maximum number of subscribers, $\overline{m}$, in the presence of disadoption is lower than the market potential, *m*. Since customers are constantly leaving, the service cannot exploit the real market potential, *m*. Rather it approaches an *effective market potential*, $\overline{m}$, which is smaller than *m* and decreases with increases in the disadoption rate. To increase this effective market potential and make it closer to the real one, firms have to invest in reducing disadoption—for example, per our calculation, the online banking industry gains about 8.9 million additional subscribers if it reduces attrition from 16% to 5%. Thus, while the traditional CRM literature acknowledges the benefits of reduced disadoption in increasing the lifetime value of a single customer through *retention*, our model illustrates the additional gains that come from the *acquisition* of customers.

The existence of an effective market potential raises the issue of interpretation of that potential: it can be viewed either as the number of consumers who will ever try the product or as the level of market saturation. When there is no disadoption, the two interpretations coincide. However, in the presence of disadoption these are two different constructs: *m* is defined as the number of people who will ever potentially try the service. The level of market saturation is $\bar{m}$, the effective market potential, which is always lower because of attrition.

Given the sensitivity of the diffusion process to levels of disadoption, one might wonder about the extent to which ignoring attrition when modeling service growth biases the parameter values. Assume a growing service category in which the diffusion parameters are estimated from continuous service (e.g., subscriber) data with a disadoption rate δ. As we have shown earlier, the penetration curve described in Equation 2 is equivalent to the Bass curve with $\bar{p}$, $\bar{q}$, and $\bar{m}$ instead of *p*, *q*, and *m*. Therefore, a researcher who estimates the parameters using the Bass function aims to estimate *p*, *q*, and *m* but actually estimates $\bar{p}$, $\bar{q}$, and $\bar{m}$. This discrepancy leads to a bias in the parameter estimation: using the definitions of $\bar{p}$, $\bar{q}$, and $\bar{m}$, it is straightforward to show that *q* and *m* are underestimated and *p* is overestimated.

To demonstrate the magnitude of the bias, we used data from three U.S. service categories that were evaluated in previous diffusion studies: cellular phones (Krishnan, Bass, and Kumar 2000; Lilien, Rangaswamy, and Van den Bulte 2000); cable television, (Lilien, Rangaswamy, and Van den Bulte 2000); and online banking (Hogan, Lemon, and Libai 2003). For each service category we obtained historical data on the number of subscribers from industry and financial reports (10K and 10Q). We used nonlinear least-square estimates (Putsis and Srinivasan 2000) to evaluate *p*, *q*, and *m* from Equation 1.

Disadoption rates (δ) are constantly monitored by firms and by industry analysts so we treated them as exogenous. We used industry data and the literature to obtain values for disadoption levels (δ) for each industry. Based on the industry reports, we estimated a yearly attrition rate of 16.5% for the U.S cable industry of 16.5% (see, for example, annual reports by the Federal Communications Commission and Kramer (2002)) and 16% for online banking (see, for example, O'Sullivan (2000)). For the cellular phone industry, we used the average disadoption rate of 8% that we computed in our empirical analysis (to be presented shortly).

Table 1 displays the results of the empirical analysis and suggests that the potential for bias is considerable. For the services in Table 1, the average overestimation of *p* is 46%, average underestimation of *q* is 39%, and average underestimation of *m* is 30%.

TABLE 1: BIAS IN PARAMETERS FOR THREE SERVICE CATEGORIES IN THE U.S.

Category	**Year**	**Bass Model**			**Service-Growth Model**				**Bias**			R^2
		p	***q***	***m* *millions***	***dis-adoption* δ**	***p***	***q***	***m* *millions***	***p***	***q***	***m***	
Cellular phones	1984–2004	.0030 (.0024)	.364 (.012)	209.1 (4.48)	8%	.00248 (.0019)	.482 (.011)	254.8 (5.45)	21%	–24%	–18%	93.5%
Cable television	1961–2004	.0029 (.0026)	.174 (.0026)	74.7 (1.8)	16.5%	.00154 (.0014)	.4044 (.006)	144.9 (3.49)	88%	–57%	–48%	49.7%
Online banking	1994–2003	.0142 (.009)	.545 (.028)	42.9 (2.3)	16%	.01101 (.0073)	.836 (.036)	55.3 (2.95)	28%	–35%	–22%	81.2%

Standard errors are in parentheses.

Another misinterpretation concerns the values of estimated *p* and *q*. Since *p* is usually regarded as influenced by the advertising policy of the firm, the biased *p* may lead firms to overestimate the influence of their advertising and, consequently, to under-invest in advertising. Similarly, the biased *q* may lead to undervaluation of the power of internal influences in the industry. Finally, neglecting attrition may be problematic when comparing penetration curves for countries or industries that differ in their disadoption rates. In such cases, differences in the curves may be related to *p, q*, and *m* so at least some of the difference is due to the different disadoption rates.

COMPETITIVE SERVICES GROWTH MODEL

We next present a model that describes the growth of a service firm and takes into account the two forms of attrition—churn and disadoption. Consider a firm that introduces a new service into a market with potential m and in which there are k competing firms. At every time t, there are customers who stop using the new service. Some of them disadopt; others defect to competitors. The attrition rate denoted by a_i consists of disadoption and churn in an additive form: *total attrition rate* (a_i) = *disadoption rate* (δ_i) + *churn rate* (c_i). Figure 1 illustrates the flow of customers to and from a focal firm in a three-firm market.

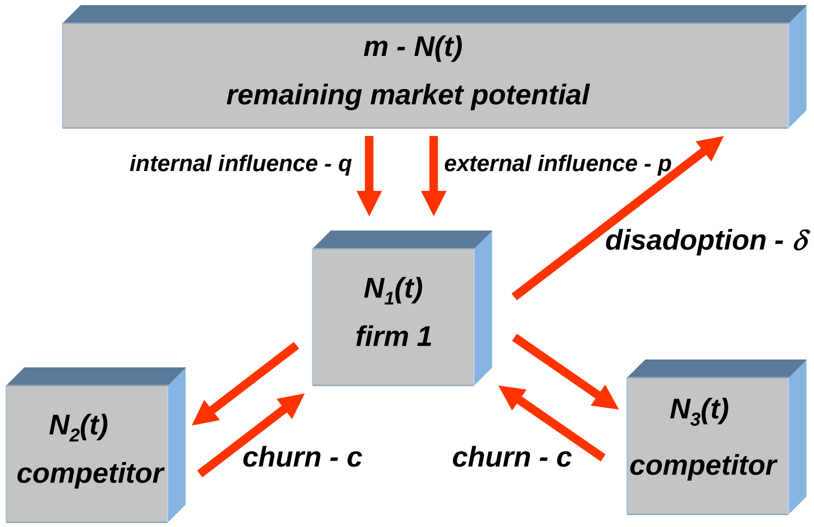
Let $N_i(t)$ be the number of subscribers of firm i at time t. The total number of subscribers in the category is $N(t) = N_1(t) + N_2(t) + \ldots + N_k(t)$. Let p_i be a parameter representing the power of external influence (advertising and other marketing efforts) while q_i represents the power of internal influence (typically word of mouth and imitation). As in conventional diffusion modeling, we assumed that word of mouth is exchanged between users and nonusers. We also assumed that internal influences are at the brand-specific level, that is, potential users join a brand of service due only to communication with existing customers of that brand. In this sense, we took the approach of Mahajan, Sharma, and Buzzell (1993) and Kalish, Mahajan, and Muller (1995).

Under the preceding assumptions, the diffusion, which is graphically illustrated in Figure 1, can be described using the following model for firm $i = 1 \ldots k$ and $a_i = c_i + \delta_i$:

$$\text{(4)} \qquad \frac{d\,N_i(t)}{d\,t} = p_i\big(m - N(t)\big) + \frac{q_i\,(1-\delta_i)N_i(t)}{m}\big(m - N(t)\big) - a_i\,N_i(t) + \sum_{j \neq i} \varepsilon_{ij} c_j N_j(t)\ .$$

The relationship between this competitive model and the category-level one in found in Appendix C (available at the *JMR* website).

Figure 1: Customer Flow to and from the Focal Firm (Firm 1) in a Three-Firm Market



Two issues are worth additional discussion at this point. First, consistent with the category-level model, at each point of time we assumed that only the $(1-\delta_i)N_i(t)$ users who did not disadopt, spread positive word-of-mouth communications. Second, we distributed churning customers among the competing firms according to the distribution parameters ε_{ij}, the share of the churn of firm j that goes to firm i. Here we assumed that the distribution of churning customers is done according to the relative number of subscribers for each firm. Thus the specification for ε_{ij} is given by $\varepsilon_{ij} = \frac{N_i}{\sum_{l \neq j} N_l} = \frac{N_i}{N - N_j}$ where $\varepsilon_{ii} = 0$ and $N = \sum_i N_i$.

The equation system presented in Equation 4 can be solved analytically under some restrictive conditions (see Appendix D - available at the *JMR* website). The solution is an S-shaped function that is similar to the penetration function of Equation 2 but with an additional term that describes the balance between a firm's effectiveness in attracting adopters and the attrition's components.

Similar to our category level model, in order to validate the identification and reliability of the model, we performed a series of simulations where we generated data with our model, estimated its parameters and observed whether our model returns the correct parameters. We randomly sampled 300 combination of a full factorial set of parameter values in typical ranges, and performed ten sets of 300 simulations each –one with clean data and the others where we added a random noise to the data. Each point of data was perturbed with a normally distributed noise term, whose standard deviation determined the noise level. We checked several noise levels, and two noise generation mechanisms – one where the noise term had a fixed standard deviation (normalized as a percentage of market potential), and relative noise, where the standard deviation of the noise term was proportional to the value of the data point. The simulation results indicate that even in the high noise level, the large majority of the parameters where correctly estimated. For example in a (relative) 6 percent noise level, 96% of the parameters were not significantly different from the original parameters at the 5% level (see Appendix B).

THE CUSTOMER EQUITY OF COMPETITIVE FIRMS: THEORY

In this section, we present an application of increasing interest among researchers and practitioners: calculating the customer equity of firms. Customer equity is the sum of the *customer lifetime value* (CLV) of each of the firm's customers (Rust, Zeithaml, and Lemon 2000; Kumar and George 2007; Villanueva and Hanssens 2007). This equity can be used, for example, as an objective function for determining the effectiveness of a firm's marketing mix and service activities when optimizing the trade-off between investing in customer acquisition versus retention or when examining the effect of operational measures such as satisfaction or attrition on the firm's long-term profitability.

One can see two basic approaches to customer equity measurement (see Kumar and George (2007) for a comprehensive review). In a disaggregate-level approach, the value of each of the firm's customers is computed individually and then summed. It fits well in cases in which the supply of individual-level customer data is rich. Under an aggregate-level top-down approach (Gupta, Lehman, and Stuart 2004; Berger and Nasr 1998; Blattberg, Getz, and Thomas 2001), firms use segment-level or firm-level data to compute the average lifetime value of a customer and multiply that average value by the number of customers to arrive at the customer equity. Given the nature of our service growth model, we chose to use the aggregate approach.

While initial approaches to customer equity that focused on profit were calculated using existing customers (Blattberg, Getz, and Thomas 2001), later work defined customer equity as the discounted sum of profits from present *and future* customers (Rust, Lemon, and Zeithaml 2004). Indeed, for growing firms the contribution of future customers to equity can be a significant part of the firm's overall equity, thus requiring estimating expected growth in the number of customers.

In the first attempt to rigorously examine the customer equity of a growing service firm, Gupta, Lehmann, and Stuart (2004) suggested an aggregate-level method for calculating the customer equity of growing service firms based on publicly available data such as the number of subscribers, margins, and retention rates (see also Gupta and Lehmann (2005)). For firm i, let the acquisition cost of a single customer be denoted by $cost_i$ and the lifetime value of a single existing customer by CLV_i; $N_i(t)$ is the number of customers of firm i at time t while $n_i(t)$ is the number of customers who join during period t. Finally, let the discount rate be denoted by ρ. Customer equity of firm i at time t is given by

(5) $$Customer \quad Equity_i(t) = N_i(t) \cdot CLV_i + (CLV_i - cost_i) \int_{s=t}^{\infty} n_i(s) \cdot e^{-\rho(s-t)} ds \, .$$

The first term on the right-hand side of Equation 5 is the contribution of the existing customer base and the second term is the summation over all future customer cohorts discounted according to the time difference between the starting point *t* and the beginning of the revenue stream from the customer. Hence, customer attrition can have a *dual effect* on customer equity: First, attrition influences the individual customer's lifetime value. Second, following our discussion in the previous section, it affects the shape of the diffusion curve expressed by $n_i(t)$.

The value of Equation 5 depends on the functional shape of the growth curve. Gupta, Lehmann, and Stuart (2004) applied their model to data for the number of individuals who ever adopted the product and not for the number of current subscribers; therefore, their work did not relate explicitly to attrition or to competitive effects. We were interested in capturing the influence of both attrition and competition; therefore, we applied Equation 5, where $n_i(t)$ is derived from the competitive services growth model in Equation 4. Note that, with some restrictive conditions, Expression 5 with the penetration function of Equation 4 can be formally calculated to yield a solution involving the Gauss ${}_2F_1$ hypergeometric function (available from the authors upon request).

In the following subsection we demonstrate the dual effect of attrition on customer equity. We calculate the customer equity of seven service firms using the growth function of the services growth model and compare the results to the stock market valuation of these firms (see Gupta, Lehmann, and Stuart (2004)). This comparison is of interest to both finance and marketing researchers since it illustrates the importance of a model that explicitly considers attrition of customers and shows to what extent and in which cases the stock market valuation agrees with or deviates from the straightforward customer equity approach presented here.

THE CUSTOMER EQUITY OF COMPETITIVE FIRMS: EMPRICAL ESTIMATION

Calculating equity in a competitive scenario requires a comprehensive set of data on market evolution. Since it involves estimation of the diffusion parameters for both the focal firm and its competitors, historical subscriber data is needed for all of the players in the category. More complex still is the comparison with the stock market value. The firm must be public and it should operate and be traded in a single competitive market.

Thus there are several limitations on the types of firms that can be used for our study. An example of an industry in which attrition plays a dominant role is cellular phone service providers. To study the cellular phone market in this respect, we used data from World Cellular Information Service (WCIS), a major data provider for this industry. Aiming to study the European cellular market, which includes sixteen countries and more than fifty operators, we found that one operator—Belgium's Mobistar—matched our requirements. A similar procedure for Pacific Asia identified one Korean operator—SK Telecom (South Korea Telecom).

Overall, we used data for seven focal firms in five markets and their main competitors, a total of thirteen firms. The focal firms were Amazon.com, Barnes&Noble.com, E*Trade, Mobistar, SK Telecom, XM Satellite Radio and Sirius Satellite Radio. Competitors of Mobistar (traded on the Brussels Stock Exchange) are Belgacom and BASE. Competitors of SK Telecom (traded on the New York Stock Exchange (NYSE)) are KT Group and LG Telecom. The two players in the U.S. satellite radio market are XM Radio and Sirius; both are traded on the NASDAQ. In the U.S.'s fragmented market, online brokerage industry reports and E*Trade's (NYSE) analysis suggested that E*Trade's main competitors are Ameritrade and Schwab.

Amazon's case is more complex because it increasingly offers lines other than books. We compared Amazon.com (NASDAQ) with its major book competitor, the online service branch of

Barnes & Noble (Barnes&Noble.com was traded on the NASDAQ until it was purchased by Barnes & Noble in the third quarter of 2003). During the timeframe of much of our data points, book retailing comprised most of Amazon's revenue and Barnes&Noble.com was considered to be Amazon's main competitor in the book market (Filson 2004; Mutter 2003). Nevertheless, Amazon's other growing business lines can impact assessments of its customer equity, which will help to explain the results from our analysis of Amazon.

We obtained customer and financial data from financial reports, 10K and 10Q forms, press releases, and WCIS. Following Gupta, Lehmann, and Stuart (2004), we took the margins and acquisition costs as averages over the preceding four quarters. Attrition rates for the satellite radio and cellular firms were taken directly from the firms' financial reports. Attrition rates for online brokerage firms were taken from a management report by Ameritrade. The rates of attrition for online book sellers were taken from Gupta, Lehmann, and Stuart (2004). Table 2 summarizes the data for each firm.

For each industry and firm *i*, the diffusion parameters p_i, q_i, m, and c were estimated by Equation 4 using seemingly unrelated, nonlinear least squares (SAS "proc model" with SUR option). We performed estimations for each firm within an industry simultaneously. Recall that overall attrition rates are constantly monitored by firms and that we could therefore treat them as exogenous. Although the model in Equation 4 allows for different attrition and churn rates among firms, in our analysis the attrition and churn rates (and therefore the disadoption rates) were taken as identical among competitors and equal to that of the focal firm. This was necessary because complete attrition data were available for the focal firms but not for the competitors. Moreover, we saw from trade publications and from the limited data available that attrition rates of firms within the same industry are quite similar. In the case of the cellular and satellite radio firms, we had more comprehensive attrition data for the nonfocal firms so we also ran the analysis by allowing the attrition and churn rates to vary among the competitors. The results of this additional analysis in terms of the other parameters were similar. However, almost all of the disadoption parameters were found to be nonsignificant.

TABLE 2: DESCRIPTIVE DATA FOR THE SEVEN FOCAL FIRMS

Focal Firm	*Competitor(s)*	*Data Period*		*Customers (millions)*	*Quarterly Margin per Customer ($)*	*Acquisition Cost per Customer ($)*	*Annual Attrition*
		From	***To***				
Amazon.com (USA)	Barnes&Noble.com	Dec. 1996	Mar. 2005	45.3	6.5	9.0	30%
Barnes&Noble.com (USA)	Amazon.com	Sep. 1997	Dec. 2004	21	1.0	4.6	30%
E*Trade (USA)	Ameritrade Charles Schwab	Dec. 1997	Mar. 2005	3.6	59.3	331.6	5%
Mobistar (Belgium)	Belgacom BASE	Jan. 1996	Dec. 2004	2.8	109.1	181.8	23%
SK Telecom (South Korea)	KT Group LG Telecom	Jan. 1984	Mar. 2005	19.0	77.6	144.6	27%
XM Satellite Radio (USA)	Sirius	Sep. 2001	Dec. 2006	7.6	20.9	128.0	18.3%
Sirius (USA)	XM Satellite Radio	Mar. 2002	Dec. 2006	6.0	24.4	128.0	18.3%

Data on number of customers, quarterly margins, and acquisition costs are the latest available for each firm, except for Barnes&Noble.com, for which quarterly margins and acquisition costs are for September 2003—the time of acquisition (after that date, the firm no longer reported that data).

Note that for the cellular operators the differences in rates between churn and overall attrition imply that, consistent with the managerial intuition in this industry, most of the attrition is churn. The disadoption rates are 7% to 8%, which is consistent with the average value over time for cellular disadoption. In satellite radio, 63% of the attrition is disadoption (quarterly churn of 1.75% translates to annual churn of 6.8% comprising of 37% of attrition), which is typical of early development of technological industries.

Table 3 presents the parameter estimations for the thirteen firms based on Equation 4.

TABLE 3: PARAMETER ESTIMATIONS BASED ON EQUATION 4

Category	***Firm***	p_i	q_i	***churn rate c***	***m***	***R Square***
Online Books#	Amazon	.00494**	0.178**	2.3%**	137.8**	22.5%
	B&N.com	.00077	.143**			42.4%
Online Brokerages#	E*Trade	.00255	.348**	1.0%	11.4**	50.6%
	Schwab	.0209**	.0807*			52.8%
	Ameritrade	.0014	.399**			87.8%
Cellular Belgium	Belgacom	.00127	.999**	16.1%**	9.2**	90.2%
	Mobistar	.00503*	.698**			83.8%
	Base	.01903*	.999*			44.3%
Cellular Korea	SK Telecom	.00373	.81**	18.0%*	45.7**	61.1%
	KT Group	.0573**	.217			89.0%
	LG Telecom	.0279**	.06			78.2%
Satellite Radio#	XM Radio	.00292**	.159**	1.75%	44.0+	53.3%
	Sirius	.00026	.272**			68.3%

#The value of *p*, *q*, and churn in Online Books, Online Brokerages and Satellite Radio refer to quarterly data.
* significant at 5%; ** significant at 1%; + Exogenous estimation.

Having estimated the parameters, we calculated customer equity for the seven focal firms using Equation 5. When calculating the customer lifetime value, we assumed a long-term planning horizon: $CLV = \sum_{t=1}^{\infty} \frac{g \cdot r^t}{(1+\rho)^t} = \frac{g \cdot r}{(1+\rho-r)} = \frac{g \cdot (1-a)}{(\rho+a)}$ where g is the gross profit margin; ρ is the discount rate, taken to be 12% as in Gupta, Lehmann, and Stuart (2004); r is the retention rate; and $a = 1 - r$. As in Gupta, Lehmann, and Stuart, we deducted the relevant corporate tax rate (38% for U.S. firms and 30% for Mobistar and SK Telecom) from the equity and used the after-tax value as a proxy for the firm's value.

To study the effects of attrition on customer equity, we compared our calculations to a competitive model that does not consider attrition. Hence, we re-estimated the parameters using

Equation 4, taking $a = c = \delta = 0$, and calculated the equity. This "no attrition" model is close in spirit to that of Kalish, Mahajan, and Muller (1995) and to the model of Krishnan, Bass and Kumar (2000) with brand-level word of mouth instead of category-level word of mouth. Table 4 presents the calculated customer equity for the seven focal firms based on expression 5, with the penetration function from Equation 4, and compares those values to calculations using the model that does not consider attrition for the last quarter of available data that we had for each firm.

The results imply that the valuation of customer equity according to the competitive services growth model is considerably higher for all firms than valuations from a model without attrition, especially for the higher attrition rates. When the attrition rate is not zero, $n_i(t) = dN_i / dt + a\, N_i(t)$, whereas a zero attrition model uses $n_i(t) = dN_i / dt$. That is, we consider the contributions of *all* of the customers who joined the service during the period. When considering only $n_i(t) = dN_i / dt$ (namely, the net difference in number of subscribers between periods), the contributions of existing customers are ignored.

TABLE 4: MARKET POTENTIAL AND CUSTOMER EQUITY

Firm	*Time*	*Actual*	*Competitive Services Growth Model*		*Competitive Model without Attrition*	
		Value ($ millions)	*Market potential (millions subs)*	*Value ($ millions)*	*Market potential (millions subs)*	*Value ($ millions)*
Amazon	Mar. 2005	18,825	137.8	5,468	86.1	1,611
B&N.com	Sep. 2003	410	137.8	330	86.1	132
E*Trade	Mar. 2005	4,923	11.4	3,769	11.1	2,816
Mobistar	Dec. 2004	5,299	9.2	4,825	8.3	1,688
SK Telecom	Mar. 2005	16,514	45.7	19,421	36.5	7,010
XM Radio	Dec. 2006	4,088	44.0	4,472	44.0	2,067
Sirius	Dec. 2006	5,705	44.0	6,818	44.0	3,591

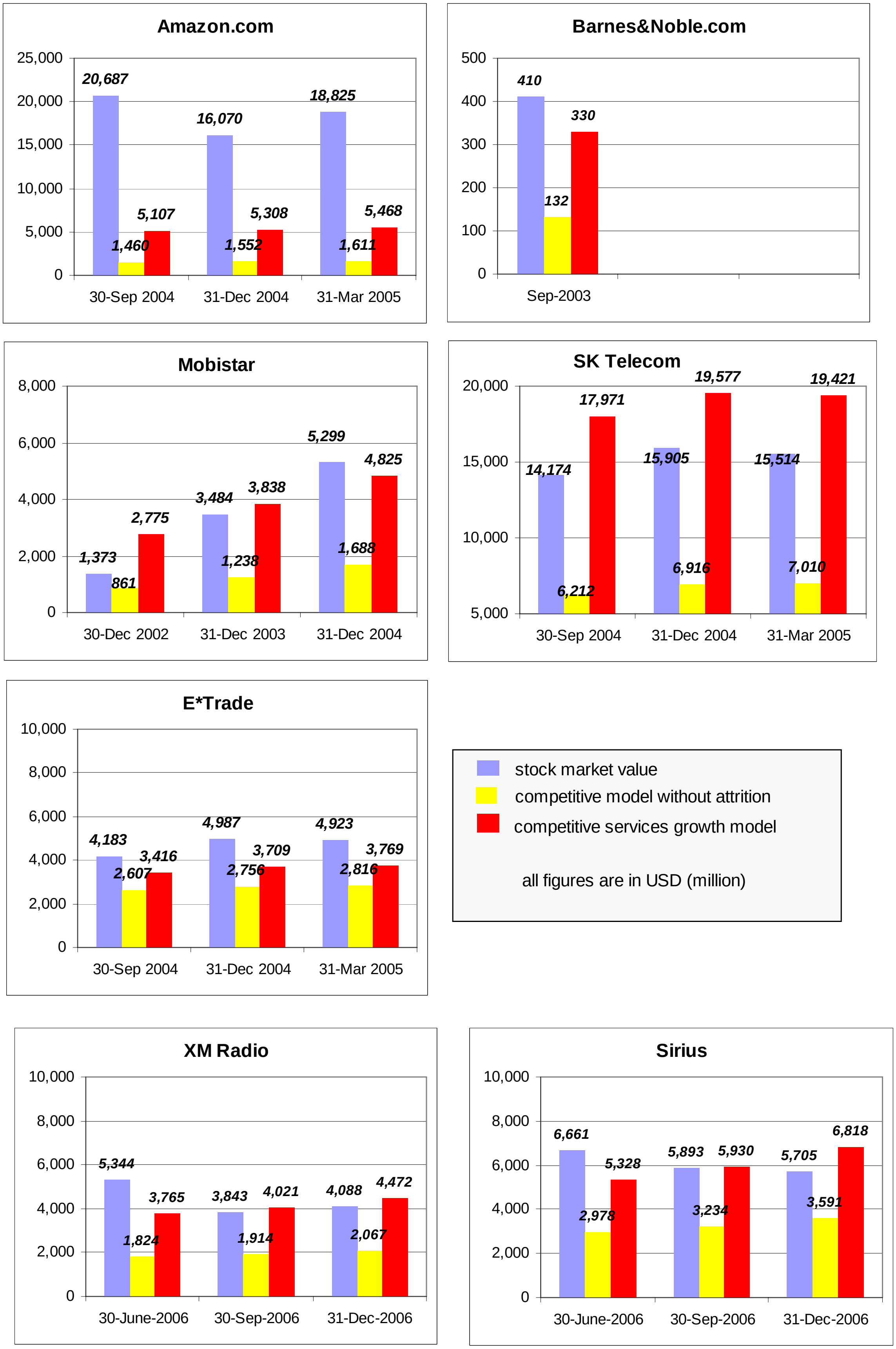
Note that adjusting the data and adding the customers who left could at least partially compensate for use of a no-attrition model. For a monopoly, the adjusted data set is the number of individuals who ever adopted the service. However, this adjustment provides only a partial compensation since it does not include the accumulated word-of-mouth contribution of these customers (who left the service). In a competitive scenario, such adjustment is problematic since one must both add the customers who left and subtract the customers who arrived from competitors. Such subtraction requires prior knowledge of the amount of churn and disadoption and, in addition, interpretation of the adjusted data is no longer clear.

Figure 2 presents a comparison between our valuation, the valuation of a model without attrition, and the average stock market value of the firms. The comparison was performed for the latest three quarters of the data in our possession. Since Mobistar provides full operational reports only once a year, we compared Mobistar for the fourth quarters of 2002, 2003, and 2004. Barnes&Noble.com was an independent public company that was traded on the NASDAQ until

2003. In the third quarter of 2003 it was purchased by Barnes and Noble. We performed the equity calculation for Barnes&Noble.com for the time of the acquisition.

Figure 2 has a number of implications. First, in all categories the competitive services growth model provides estimations that are considerably closer to the stock market values than those from the model that does not consider attrition. For six of the seven firms the customer equity estimations are remarkably close to the stock market values. If we take the latest valuation for Barnes&Noble.com, E*Trade, Mobistar, SK Telecom, XM Radio, and Sirius, we find that the average deviation of our calculated values from that of the stock market is 17.7%. The one exception is Amazon.com, which is traded at notably higher values than the ones generated by our model and is discussed in the next section.

Figure 2: Market Value and Customer-Based Valuation



This approach has several limitations. First, the stock market value may be influenced by factors other than future customer-related cash streams, such as various assets or debts the firm owns or owes. Indeed, in cases where customer-related profits do not play a major part in expected future earnings, careful adaptation should be made before comparing the two values. The second issue is that the stock market may make assumptions regarding future growth (related, for example, to firms entering new markets) that are not embedded in our model.

DISCUSSION

At the beginning of this paper, we compared the growth of competitive services to a leaking bucket. There is an inward flow of customers, new adopters and customers who switch from competitors. There is also an outward stream of customers who either disadopt the category or defect to the competition. This complex environment makes analysis of the growth of new services nontrivial; yet the ubiquity and the importance of new services makes such an analysis essential for managers wishing to understand the environment in which their services compete.

We presented a competitive service growth model that provides a platform for this analysis. Our approach is relatively straightforward and, with a few simplifying assumptions, the basic model has a closed-form solution. To demonstrate potential applications, we focused first on the category level and examined the role that category-level attrition plays in the evolution of markets for new services. We demonstrated how using a durable goods approach to study the growth of a service can considerably bias estimation of the parameters of growth.

We then moved to the brand level and used our approach to develop a method of functionally estimating the customer equity of firms. The approach presented here is the first customer equity measure that takes into account interfirm dynamics in a growing market and is especially critical to the calculation of customer equity when firms are strongly affected by both customer switching to competitors and by disadoption of the category.

As firms aim to better understand the economic value they give to their shareholders, there is increasing interest in the marketing/finance interface in general (e.g., Kumar and Petersen 2005; Hogan et al. 2002). This is the motivation for recent efforts to contrast a service firm's customer equity and stock market value (Gupta, Lehmann, and Stuart 2004). A match-up between the two is not always straightforward, especially when a firm's long-range customer value is just one of the sources of value for shareholders. For a service firm that derives value mostly from customers, the comparison is relevant and can help researchers better understand the role that customer equity plays in the perceptual value of firms.

We used our approach with seven firms in four service categories. As Figure 2 demonstrates, we found that, with the exception of Amazon.com, our model's estimates of customer equity were remarkably close to the stock market values for the six firms, with an average difference of about 18%. Interestingly, while our approach generated values that were quite close to the stock market values, a competitive model that ignored attrition yielded much lower valuations. With the necessary caution stemming from our small sample, our results suggest that attrition plays a critical part in how the stock market values service firms. This may turn out to be an important aid to those advocating proper management of customer assets as a way to increase shareholder value and, in a more general sense, as evidence of the role of marketing for service firms.

The one exception to our relatively reasonable valuations was Amazon.com, an example that likely stems from the limitation of the long-term approach we took. The value of Amazon.com's stock has been the subject of much industry-related discussion since the late 1990s, with some notable experts repeatedly claiming that it is overvalued (Damodaran 2001; Hough 2003). The main issue likely relates to assumptions about Amazon's future growth. Since Amazon.com is no longer an online bookseller and is rather a general online retailer and since much of its growth may stem from nonbook categories (some which are not retailing, such as web-based advertising), one may argue that relying on recent margins and growth patterns in this case is not realistic.

Our aim is not to be part of the specific debate regarding Amazon but to point out that customer equity approaches of the type presented here are a potentially appropriate framework for the analysis, even as a sensitivity analysis tool. For example, some analysts wonder what margin or growth rate would justify Amazon's high share price (Hough 2003). Since these are parameters in our model, analysts can use such a customer equity approach to make more informed decisions and justify their positions. Of course, it may be that Amazon's unique market reality does not fit well with the simple assumptions of the model we present here. Researchers aiming to further examine this case may need to develop extensions to our model that better take into account the case of a rapidly changing target market. This, however, is beyond the scope of this paper.

LIMITATIONS AND EXTENTIONS

The services growth model relies on a number of assumptions, mainly related to the nature of attrition. The assumptions were made to provide an analytical formulation and enable empirical estimations. However, extended models can be developed that relax some of these assumptions.

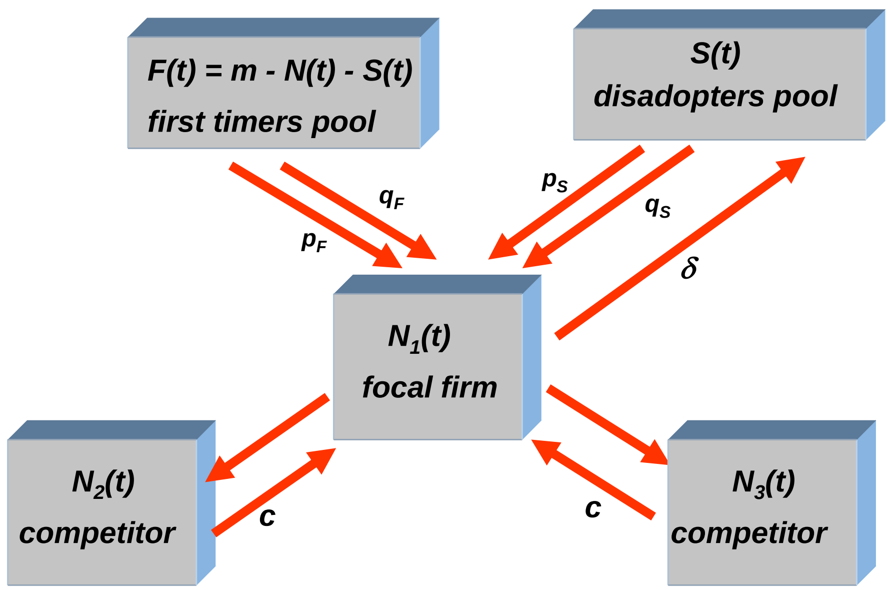
One such assumption regards the equal diffusion parameters for the re-adoption of disadopting customers compared with the acquisition of new ones. Interestingly, there is little evidence to date from which to infer the difference between the two groups precisely as there is little empirical research on customer reacquisition in general (see Thomas, Blattberg, and Fox (2004) for an exception). Theoretically, disadopters may need less information to readopt since they have already used the product and are aware of its performance. However, consumers who disadopted may be reticent to readopt without solid evidence that the second time around will be a satisfactory experience.

Relaxing this assumption theoretically can be done by splitting the pool of potential users into a pool of first-time adopters and a pool of disadopters who might eventually readopt (see Figure 3 below). However, such a model can be difficult to estimate without much richer data on customer word-of-mouth behavior. Meanwhile, the values of the external and internal influence parameters in our model represent the average communication impact of both new adopters and readopters and should be interpreted accordingly.

What is the minimal model that is necessary to abstract from the assumption of equal influence on disadopting customers new adopters? In this minimally extended model (see Figure 3), the pool of nonadopters is broken down into two subpools—first-time adopters and disadopters (of those who adopted at least once). The equations describing this model are presented in Appendix E.

This model is difficult to apply in practice, due to its large number of parameters. For three firm markets, such as the cellular industries in our sample, this model requires six more parameters on top of the eight parameters we already had with Equation 4. This can create an estimation problem, especially with a small number of data points. In order to test the reliability of this model, we performed simulations in the same procedure as described for the category level and brand-level models – that is, generating data with the model and then trying to recover the model parameters using non-linear regression. We generated clean data and noisy data in various noise levels. Our results indicate that although the model correctly estimates the true parameters for the clean data, performance deteriorates considerably when noise is added. For example, in a noise level of 4% (relative to each data point value), less than three quarters of the parameters were estimated correctly (see Appendix B - available at the *JMR* website). In addition, the average R^2 value was 40%, as compared with 78% for the simulations of Equation 4. However, this model can still be applicable for sets with large number of data points.

Figure 3: An Extended Competitive Services Growth Model



The marketing literature has, since the 1990s, been emphasizing a study of customer attrition and its implications for marketing strategies. Incorporation of customer attrition into mainstream marketing models is part of the shift in the marketing discipline from the study of marketplace exchanges as transactions to that of relationships that should be managed and examined in the long run (Agustin and Singh 2005; Morgan and Hunt 1994). To accomplish that shift, marketers must adapt the tools they use and we hope that this study can serve as a step at that direction.

Appendices: (Appendices A through D are posted at the *JMR* website).

Appendix E: The extended model

In the paper, we assumed equal probability of return of disadopting customers compared with the acquisition probability of new ones. This assumption means that while customers differ in their *p* and *q*, we consider the average values of their probabilities. Relaxing this assumption is done by splitting the pool of potential users into a pool of first-time adopters and a pool of disadopters who might eventually re-adopt (see Figure 3).

Under these assumptions, the diffusion process can be described using the following set of differential equations:

$$\frac{d\,N_i}{d\,t} = p_{iF}\left(m - N - S\right) + \frac{q_{iF}\,(1-\delta_i)N_i}{m}\left(m - N - S\right) + p_{iS}S + \frac{q_{iS}\,(1-\delta)N_i}{m}S - a_i\,N_i + \sum_{j\neq i}\varepsilon_{ij}c_j N_j$$

$$\frac{dS}{dt} = -\sum_{i=1}^{k} p_{iS}S - \sum_{i=1}^{k}\frac{q_{iS}\,(1-\delta_i)N_i}{m}S + \sum_{i=1}^{k}\delta_i N_i$$

We checked the reliability of the model using the procedure described in Appendix B (available at the *JMR* website): We randomly sampled 30 combinations of a full factorial set of parameter values in typical ranges, for the three firms case, generated data with these parameters using the above equations, and then performed ten sets of 300 simulations each – one with clean data, four with absolute noise (noise levels of 1%, 2%, 4%, and 6%), and five in various levels of relative noise (noise levels of 1%, 2%, 4%, 6%, and 10%). The estimations were performed using SAS (proc model, SUR option). The rest of the procedure precisely followed the ones we did in Appendix B. Table E.1 displays the number of estimations which were significantly different from the original parameters at the 5% level. The table implies that with clean data, the parameters are fully recovered by the model. However, when noise is added, the number of cases of wrong estimations increases substantially even in the low noise levels. In the noisy case, the estimation of many parameters received the boundary values (0 or 1), and standard errors are large. Average adjusted R-Square (averaged over all the simulations) is 40%.

Table E.1: Simulations of extended model; set size=30 simulations; 420 parameters per set

Noise type	Noise level	No of parameters significantly different from true values	Percentage
no noise	0	0	0
absolute	1%	127	30%
absolute	2%	152	36%
absolute	4%	112	26%
absolute	6%	176	42%
relative	1%	68	16%
relative	2%	95	23%
relative	4%	112	27%
relative	6%	126	30%
relative	10%	153	36%

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